

$J^P = \frac{1}{2}^+, \frac{3}{2}^+$ masses in statistical model

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Abstract

The mass formulae for the baryon octet and decuplet are calculated. These formulae are function of constituent quark masses and spin spin interaction terms for the quarks inside the baryons. The coefficients in the mass formulae is estimated by the statistical model for $J^P = 1/2^+, 3/2^+$, incorporating the contributions from “sea” containing $u\bar{u}, d\bar{d}, s\bar{s}$ pairs and gluons. The measured masses are presented and found to be matching good with some of the experimental and theoretical data.

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1. Introduction

Many calculations have been performed in the last few years with the aim of understanding the baryon mass spectrum. In order to understand the hadron spectroscopy the low energy properties should be essentially understood. A simple yet unique concept of particle physics, i.e. quark model suggest that all hadrons are made of of three quarks or a quark and an antiquark, bounded by all interactions which arise from renormalizable gauge couplings. In this paper, we will try to understand the baryonic mass spectrum from this standpoint. Many of the hadron mass splitting observed till date are produced by the splitting among the quark masses. Much research has been devoted in describing hadron masses which also includes masses of hadrons with heavy quarks, often with considerable success [1–9]. Morpurgo [10], by the use of field theory and the non relativistic quark model, gave a general parameterizations of the magnetic moments and masses of baryon octet and decuplet. His general parametrization method (GPM), derived from the QCD Lagrangian, expresses the mass operators in terms of flavor-dependent terms proportional to powers of the strange-quark projection operator P^λ and non relativistic appearing products of Pauli spin operators σ .

Several authors have studied the masses of heavy hadrons in the non-relativistic quark model with the inclusion of spin and flavor dependant hyperfine interaction between two quarks and between a quark and an antiquark and many other different techniques [12]. Study of baryon mass spectrum has been a subject of increasing interest due to related experiments at Fermilab, CERN etc. [13]. Also several models including non relativistic quark model (NRQM) [2, 4], chiral perturbation theory (ChPT) [14], hyper central model [15] have evaluated the data of light and heavy baryon mass spectrum having nice agreement with experimental information available. C.

P. Singh [2] et. al., calculated the masses of charmed and b-quark hadrons in the non relativistic additive quark model with the inclusion of spin and flavor dependant hyperfine interaction between two quarks and a quark and an antiquark, which was in agreement with the available data.

Masses of heavy and light baryons are computed using the equation $\text{Mass} = M_{quark} + \text{Spin-Spin Correction}$, where M_{quark} is the mass obtained using scalar quarks and the spin-spin correction term. In order to understand mass spectrum, spin-spin interaction term has a pivotal role in the mass formulae. This term is short range, in the sense that interaction energy associated with spin-spin coupling of quarks decreases with distance as α_S^{-N} . In general, it is only the short range forces between the quarks which are spin dependant. On the other hand, the lattice is good for the long distance physics, the lattice cutoff can spoil the short-distance physics. In this note, we consider the baryon masses in the statistical picture where the short range forces are focussed. It is worthy to mention that we are considering S-wave quark-antiquark systems here. The parameters required for these calculations are completely determined by previously studied properties, using the wave functions, so the results are entirely predictive. The spin-spin interaction contribution is also responsible for $\Sigma - \Lambda$ mass splitting in case of baryons [11]. The reason for this splitting is that, in case of Σ^0 , the quarks are in symmetric state which means that quark pair (u,d) must also be symmetric in spin state, such that $\vec{S}_u \cdot \vec{S}_d = \frac{1}{4}$. For Λ , the quark pair (u,d) has isospin zero which means total spin is zero (antisymmetric state), such that $\vec{S}_u \cdot \vec{S}_d = -\frac{3}{4}$. The ‘‘hyperfine’’ splittings for $\Sigma - \Lambda$ are related by:

$$\Sigma - \Lambda = \frac{2}{3}(1 - \frac{m_u}{m_d})(\Delta - N)$$

The scheme of the the present manuscript is as follows: Section II begins with a brief review of construction of decuplet wave function with sea. Section III presents the explanation of model used i.e. statistical model and principle of detailed balance. Mass formulae are defined in section IV followed by calculation of the masses in section V. Discussion and conclusion is presented in Section VI.

2. Preliminaries

The structure of hadron constitutes two parts i.e valence part (qqq) and other is sea part which consist of quark-antiquark pairs muticonnected through gluons [16–18]. A q^3 state in the baryon are in the 1, 8 and 10 color states which means that sea should also be in corresponding states to form a color singlet baryon. The valence part of the hadronic wave function can be written as:

$$\Psi = \Phi(|\phi\rangle|\chi\rangle|\psi\rangle)(|\xi\rangle) \quad (2.1)$$

where $|\phi\rangle, |\chi\rangle, |\psi\rangle$ and $|\xi\rangle$ denote flavor, spin, color and space q^3 wave functions and their contribution yields antisymmetric total wave function. Here, spatial part ($|\xi\rangle$) is symmetric under the exchange of any two quarks for the hadrons and therefore the flavor-spin-color part $\Phi(|\phi\rangle|\chi\rangle|\psi\rangle)$ should be antisymmetric in nature such that when combined with ($|\xi\rangle$) gives antisymmetric total wave function. To show an active participation of sea, a relevant wave function is written with valence and quark gluon Fock states. Sea considered here is in S-wave state with spin (0,1,2) and color (1,8, $\overline{10}$) and is assumed to be flavorless. Let $H_{0,1,2}$ and $G_{1,8,\overline{10}}$ denote spin and color sea wave functions, which satisfy $\langle H_i | H_j \rangle = \delta_{ij}$, $\langle G_k | G_l \rangle = \delta_{kl}$. In this approach we have assumed a sea

to be consisting of three gluons. All possible combinations of valence q^3 and sea wave functions which can yield spin 1/2 (3/2), flavor octet (decuplet) and color singlet state thereby maintaining the anti symmetrization of the total baryonic wave function are [19, 20]:

$$\begin{aligned} \text{Octet} = & \Phi_1^{(1/2)} H_0 G_1, \Phi_8^{(1/2)} H_0 G_8, \Phi_{10}^{(1/2)} H_0 G_{\overline{10}}, \Phi_1^{(1/2)} H_1 G_1, \Phi_8^{(1/2)} H_1 G_8, \\ & \Phi_{10}^{(1/2)} H_1 G_{\overline{10}}, \Phi_8^{(3/2)} H_1 G_8, \Phi_8^{(3/2)} H_2 G_8 \end{aligned} \quad (2.2)$$

$$\text{Decuplet} = \Phi_1^{(3/2)} H_0 G_1, \Phi_1^{(3/2)} H_1 G_1, \Phi_8^{(1/2)} H_1 G_8, \Phi_1^{(3/2)} H_2 G_1, \Phi_8^{(1/2)} H_2 G_8 \quad (2.3)$$

The total flavor-spin-color wave function of a spin up baryon octet (decuplet) consisting of three valence quarks and a sea component can be written as:

$$\begin{aligned} \text{Octet} = |\Phi_{1/2}^{(\uparrow)}\rangle = & \frac{1}{N} [\Phi_1^{(1/2\uparrow)} H_0 G_1 + a_8 (\Phi_8^{(1/2)} \otimes H_0)^\uparrow G_8 + a_{10} \Phi_{10}^{(1/2\uparrow)} H_0 G_{\overline{10}} \\ & b_1 (\Phi_8^{(1/2)} \otimes H_1)^\uparrow G_1 + b_8 (\Phi_8^{(1/2)} \otimes H_1)^\uparrow G_8 + b_{10} (\Phi_{10}^{(1/2)} \otimes H_1)^\uparrow G_{\overline{10}} \\ & + c_8 (\Phi_8^{(3/2)} \otimes H_1)^\uparrow G_8 + d_8 (\Phi_8^{(3/2)} \otimes H_2)^\uparrow G_8] \end{aligned} \quad (2.4)$$

where

$$N^2 = 1 + a_8^2 + a_{10}^2 + b_1^2 + b_8^2 + b_{10}^2 + c_8^2 + d_8^2 \quad (2.5)$$

$$\begin{aligned} \text{Decuplet} = |\Phi_{3/2}^{(\uparrow)}\rangle = & \frac{1}{N} [a_0 \Phi_1^{(3/2\uparrow)} H_0 G_1 + b_1 (\Phi_1^{(3/2)} \otimes H_1)^\uparrow G_1 + \\ & b_8 (\Phi_8^{(1/2)} \otimes H_1)^\uparrow G_8 + d_1 (\Phi_1^{(3/2)} \otimes H_2)^\uparrow G_1 + \\ & d_8 (\Phi_8^{(1/2)} \otimes H_2)^\uparrow G_8] \end{aligned} \quad (2.6)$$

where

$$N^2 = a_0^2 + b_1^2 + b_8^2 + d_1^2 + d_8^2 \quad (2.7)$$

Here, N is the normalization constant. The first three terms in the eq. (2.4) are obtained by combining q^3 wave function with spin 0 (scalar sea) and next three terms are obtained by coupling q^3 with spin 1 (vector sea). The final two terms are the result of coupling with spin 2 (tensor sea). Similarly, the first term in eq. (2.6) is obtained by combining the q^3 wave function with spin 0 (scalar sea) and the next two terms are obtained by coupling q^3 with spin 1 (vector sea) and the final two terms are the result of the coupling with spin 2 (tensor sea). The details of all the terms of wave function in equation (2.4) and (2.6) can be found in references [19, 20]. All the coefficients in the above wave functions are determined statistically from the flavor, spin and color probabilities for the study of low energy properties of hadrons.

3. Principle of detailed balance and Statistical Model

The main idea of the detailed balance model, proposed by Zhang et. al. [22, 23], is that it assumes the proton as a bag of quark-gluon gas in dynamical balance, where partons keep combining and

splitting through processes such as $g \Leftrightarrow q\bar{q}, g \Leftrightarrow gg, q \Leftrightarrow qg$. This model was proposed to study the $\bar{d}-\bar{u}$ asymmetry in nucleon and it was found that the detailed balance model gives $\bar{d}-\bar{u}=0.124$ [24], which was in agreement with the E866/NuSea result of 0.118 [25]. This good agreement indicates that the principle of detailed balance plays an important role in the structure of proton. The method was also extended to pions [26] and the nucleon spin structure [27]. Later, the strange content of the proton was also calculated, using the balance model under the equal probability assumption.

The model begins with the valance quark structure of the proton without any parameters. In this picture, while $d\bar{d}$ and $u\bar{u}$ sea quark-antiquark pairs are produced by gluon splitting with equal probability, the reverse process, i.e., the annihilation of antiquarks with their quark partners into gluons, is not flavor symmetric due to excess of u quarks over d quarks in the proton. In general, detailed balance principle demands that the exchange between any two states should balance each other, which can be expressed as:

$$n_{A \rightarrow B} = n_{B \rightarrow A} \quad (3.1)$$

where A and B are the states. Hadron is treated to be consisting of complete set of quark gluon Fock states and can be expressed in expanded form as:

$$|Baryon\rangle = \sum_{i,j,l,k} C_{i,j,l,k} |q^3\rangle, (i,j,l), (k)\rangle \quad (3.2)$$

where q^3 represents the valance quarks of the baryon, i is the number of $u\bar{u}$ pairs, j is the number of $d\bar{d}$ pairs, l is the number of $s\bar{s}$ pairs and k is the number of gluons in sea. The probability to find a quark-gluon Fock states in the baryon system is:

$$\rho_{i,j,l,k} = |C_{i,j,l,k}|^2, \quad (3.3)$$

and $\rho_{i,j,l,k}$ satisfies the normalization condition,

$$\sum_{i,j,l,k} \rho_{i,j,l,k} = 1 \quad (3.4)$$

The transition probabilities in flavor space for various Fock states have already been determined for nucleon and hyperons containing strange sea and can be found in references [23, 28]. This model, till date, has been able to explain flavor asymmetry, magnetic moments, spin distribution, semileptonic decays of nucleon for octet and decuplet particles [20, 21, 28, 29]. We will, here, use this model to calculate the mass splittings of octet and decuplet particles. Also, it is worthy to mention that the quarks and gluons in the Fock states are the intrinsic partons of hadrons as they are non-perturbatively multiconnected to valance quarks. The “intrinsic” sea quarks and gluons survive over a long lifetime within hadronic bound states whereas the “extrinsic” sea quarks and gluons only exist for a short time. The extrinsic partons in the Fock states are generated from QCD hard bremsstrahlung or gluon splitting as part of the lepton scattering interaction.

The Statistical model [27] is used in our formalism to calculate the masses of octet and decuplet members by assuming hadrons as an ensemble of three valance quarks and sea containing various quark-gluon Fock states. The main feature of the statistical model is that it does not requires any additional input parameters which proves its physical simplicity, that have made an amazing success in describing parton distribution functions for nucleons. A more general description

of this model was provided by J.P Singh et. al., where in addition to flavor, each Fock state has some definite spin and color quantum number with a specific symmetry property [27]. Here, it is worthy to mention that all the above listed properties were directly linked to probabilities of each Fock state in definite spin, color, flavor space quantum numbers. The different possible states in spin, flavor and color, for two gluons can be written as:

$$\begin{aligned}\textbf{Spin} : gg : 1 \otimes 1 &= 0_s \oplus 1_a \oplus 2_s \\ \textbf{Color} : gg : 8 \otimes 8 &= 1_s \oplus 8_s \oplus 8_a \oplus 10_a \oplus \overline{10}_a \oplus 27_s \\ \textbf{Flavor} : 3 \otimes 3 \otimes 3 &= 1_a \oplus 8_{ms} \oplus 8_{ma} \oplus 10_s\end{aligned}$$

The decuplet is symmetric in flavor, singlet antisymmetric and the two octets have mixed symmetry. Subscripts s and a denotes symmetry and antisymmetry on combining the states. Total antisymmetry of the baryon should be kept in mind while combining the valence and sea part. In this model, all $n'_{\mu,\nu}$ s are calculated from multiplicities of each Fock state in spin and color space. These multiplicities are expressed in the form of $\rho_{p,q}$ where relative probability for core part should have spin p and sea to have spin q such that the resultant should come out as 1/2 (3/2). Similar probabilities could be calculated for color space which yields color singlet state. Calculation of these probabilities helps to find common factor “c” for every combination of valence and sea which is multiplied with multiplicity factor (n) for each Fock state. The common parameter “c” can be calculated from the various Fock states derived from the principle of detailed balance. Each unknown parameter in the equation of wave function will have a particular value of $\sum n_{\mu\nu} c_{sea}$ depending on the Fock state [20]:

$$a_0^2 = (n_{01} c_{sea})_{|gg\rangle} + (n_{01} c_{sea})_{|u\bar{u}g\rangle} + (n_{01} c_{sea})_{|d\bar{d}g\rangle} + (n_{01} c_{sea})_{|s\bar{s}g\rangle} + \dots \quad (3.5)$$

$$b_1^2 = (n_{11} c_{sea})_{|gg\rangle} + (n_{11} c_{sea})_{|u\bar{u}g\rangle} + (n_{11} c_{sea})_{|d\bar{d}g\rangle} + (n_{11} c_{sea})_{|s\bar{s}g\rangle} + \dots \quad (3.6)$$

$$d_1^2 = (n_{21} c_{sea})_{|gg\rangle} + (n_{21} c_{sea})_{|u\bar{u}g\rangle} + (n_{21} c_{sea})_{|d\bar{d}g\rangle} + (n_{21} c_{sea})_{|s\bar{s}g\rangle} + \dots \quad (3.7)$$

$$\dots\dots\dots \quad (3.8)$$

Combinations for other unknown parameters can be written in a similar way. A detailed information for calculation of all the parameters using statistical model is being provided in references [19,20]. The calculations for probabilities are done in two ways i.e C model and D model. Model D assumes that a sea containing a large number of gluons has relatively smaller probabilities and hence their higher multiplicities have been suppressed over the rest of valence particles with limited quarks. Sea with larger color multiplicity has less probability of survival due to larger possibility of interaction. Model D is assumed to be a special case of Model C. The parametrization of model D, can be achieved by assuming that probability of a state is inversely proportional to multiplicity of state in both spin and color states. Therefore, the new probabilities are additional to previous found probabilities factors.

4. Mass Formulae

The hadronic mass spectrum is an essential ingredient in theoretical study of the physics involving strong interactions. Mass spectrum of mesons and baryons are studied in various models and aspects. The hadron mass formula is discussed in the quark-counting aspect which shows that the “free” quark picture gives the GellMann-Okubo formula [30]. In quark models, the baryon

octet and decuplet are bound state of three-quark states (q^3) and there are various different model calculations for their masses. Empirical mass formulae [31] for the baryon octet and decuplet are the functions of one integer variable assigned to each member of the baryon sets and charge state of the baryon. These formulae are independent of any specific model. Further, the gross features of simple quark model has helped in unraveling the detailed properties of mass spectra of baryons and mesons. For example, how we determine the different masses of $\Lambda^{(\frac{1}{2})}$ (1115.683), $\Sigma^{0(\frac{1}{2})}$ (1192.642) and $\Sigma^{*0(\frac{3}{2})}$ (1382.8), despite having same quark content. The difference in spin spin interactions among quarks is the answer to this query, as explained in section 1. We assume, that mass of hadron arises from the constituent quark masses plus the spin-spin interaction energies of quarks for a meson and a baryon, can be written as [32]:

$$M_{meson} = a_m + m_i + m_j + bm_0^2 \frac{s_i \cdot s_j}{m_i m_j} \quad (4.1)$$

$$M_{baryon} = a_b + \sum_i m_i + \frac{bm_0^2}{3} \sum_{i < j} \frac{s_i \cdot s_j}{m_i m_j} \quad (4.2)$$

where a_m , a_b , b are parameters with the dimension of mass and m_i , m_j are the masses of respective quarks (antiquarks), m_0 is a scale factor which we shall take to be the mass of the lightest quark, i.e. $m_0 = m_u, m_d$. The spin dependent term includes a contribution of each pair that is proportional to the expectation value of $s_i \cdot s_j$ and inversely proportional to the product of the constituent quark masses $m_i m_j$. The spin-spin interaction used in the mass operator may be interpreted as the interaction between color magnetic moments proportional to $g/2m$ in analogy with Dirac magnetic moment, g is the strong-coupling constant of gluons with quarks. With the help of the wave function of baryon octet and decuplet, described in previous section, eqns. (3.1) and (3.2) give the masses of the hadrons in terms of the parameters. To evaluate equation (4.2) for each baryon, we need to find the expectation values for spin operators for each quark pair within the respective baryon. The spin interaction term we need to find for these baryons, which are made of up of, say, u , d , s is,

$$\frac{\vec{S}_u \cdot \vec{S}_d}{m_u^2} + \frac{\vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s}{m_u m_s} \quad (4.3)$$

Here, $m_u = m_d$. The eigen values for $s_u \cdot s_d$ are $1/4$ and $-3/4$ for spin triplet ($I=1$) and singlet ($I=0$) states respectively, results in evaluation of other terms by: Consider $\vec{J} = \vec{S}_u + \vec{S}_d + \vec{S}_s$

$$\text{Then, } \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s = \frac{1}{2}[J^2 - (S_u^2 + S_d^2 + S_s^2)] - \vec{S}_u \cdot \vec{S}_d \quad (4.4)$$

$$\begin{aligned} \text{For octet, } \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s &= -1(\text{symmetric}) \\ \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s &= 0(\text{antisymmetric}) \end{aligned} \quad (4.5)$$

$$\begin{aligned} \text{For decuplet, } \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s &= \frac{1}{2}(\text{symmetric}) \\ \vec{S}_u \cdot \vec{S}_s + \vec{S}_d \cdot \vec{S}_s &= \frac{3}{2}(\text{antisymmetric}) \end{aligned} \quad (4.6)$$

These values are applicable for all the baryon octet and decuplet particles relative to their quark content. The mass operator given in equation (4.2) is applied to the terms of the wave function in equation (2.4) and (2.6). The eigen values of spin operator given in equation (4.5) and (4.6) is then used to obtain the relations in terms of parameters with the dimension of mass. Mass relations thus obtained are displayed in table 1 and 2 for $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ states.

Term	After applying operator on each term
$a_0 \Phi_1^{(1/2)} H_0 G_1$	$a_0(a_b + \sum_i m_i + \frac{bm_0^2}{3}(\frac{1}{\sqrt{2}})[\frac{1}{4m_i m_j} - \frac{1}{m_i m_j} - \frac{3}{4m_i m_j}]))$
$a_8 \Phi_8^{(1/2)} H_0 G_8$	$a_8(a_b + \sum_i m_i + (-\frac{bm_0^2}{3}[\frac{1}{4m_i m_j} - \frac{1}{m_i m_j}]))$
$a_{10} \Phi_{10}^{(1/2)} H_0 G_{\overline{10}}$	$a_{10}(a_b + \sum_i m_i + \frac{bm_0^2}{3}(\frac{1}{\sqrt{2}})[- \frac{3}{4m_i m_j} - \frac{1}{4m_i m_j} + \frac{1}{m_i m_j}]))$
$b_1(\Phi_1^{(1/2)} \otimes H_1)G_1$	$b_1(a_b + \sum_i m_i + \frac{bm_0^2}{3}(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}})[- \frac{3}{4m_i m_j} + \frac{1}{4m_i m_j} - \frac{1}{m_i m_j}]))$
$b_8(\Phi_8^{(1/2)} \otimes H_1)G_8$	$b_8(a_b + \sum_i m_i + \frac{bm_0^2}{3}(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{12}})[\frac{1}{4m_i m_j} - \frac{1}{m_i m_j} + \frac{3}{4m_i m_j}] + (\frac{1}{\sqrt{12}} - \frac{1}{\sqrt{6}})[- \frac{3}{4m_i m_j} + \frac{1}{4m_i m_j} - \frac{1}{m_i m_j}]))$
$b_{10}(\Phi_{10}^{(1/2)} \otimes H_1)G_{\overline{10}}$	$b_{10}(a_b + \sum_i m_i + \frac{bm_0^2}{3}(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}})[\frac{1}{4m_i m_j} - \frac{1}{m_i m_j} + \frac{3}{4m_i m_j}]))$
$c_8(\Phi_8^{(3/2)} \otimes H_1)G_8$	$c_8(a_b + \sum_i m_i + \frac{bm_0^2}{3}(\frac{1}{2} - \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{12}})[\frac{3}{4m_i m_j}]))$
$d_8(\Phi_8^{(3/2)} \otimes H_2)G_8$	$d_8(a_b + \sum_i m_i + \frac{bm_0^2}{3}(\frac{1}{\sqrt{5}} - \sqrt{\frac{3}{20}} + \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{20}})[\frac{1}{4m_i m_j} - \frac{1}{m_i m_j}]))$

Table 1: Derived mass relation for $J^P = \frac{1}{2}^+$ particles. Here, m_i, m_j are the constituent quark masses of the respective quarks in the baryon, a_b, b are the parameters with the dimension of mass and m_0 is a scale factor equal to mass of the lightest quark, i.e $m_0 = m_u, m_d$.

Term	After applying operator on each term
$a_0 \Phi_1^{(3/2)} H_0 G_1$	$a_0(a_b + \sum_i m_i + \frac{bm_0^2}{3}[\frac{1}{4m_i^2} + \frac{1}{2m_i m_j}])$
$b_1 \Phi_1^{(3/2)} H_1 G_1$	$b_1(a_b + \sum_i m_i + \frac{bm_0^2}{3}[\sqrt{\frac{3}{5}}[\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}] - \sqrt{\frac{2}{5}}[\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}]])$
$b_8 \Phi_8^{(1/2)} H_1 G_8$	$b_8(a_b + \sum_i m_i + \frac{bm_0^2}{3}[\sqrt{\frac{3}{5}}[\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}] - \sqrt{\frac{3}{5}}[\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}]])$
$d_1 \Phi_1^{(3/2)} H_2 G_1$	$d_1(a_b + \sum_i m_i + \frac{bm_0^2}{3}[\sqrt{\frac{1}{5}}[\frac{1}{4m_i^2} + \frac{1}{2m_i m_j}] - \sqrt{\frac{2}{5}}[\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}] + \sqrt{\frac{2}{5}}[\frac{-3}{4m_i^2} + \frac{3}{2m_i m_j}]])$
$d_8 \Phi_8^{(1/2)} H_2 G_8$	$d_8(a_b + \sum_i m_i + \frac{bm_0^2}{3}[\sqrt{\frac{1}{10}}[\frac{1}{m_i^2} - \frac{1}{m_i m_j}] - \sqrt{\frac{2}{10}}[\frac{1}{m_i^2} - \frac{1}{m_i m_j}] + \sqrt{\frac{2}{10}}[\frac{1}{m_i^2} - \frac{1}{m_i m_j}]])$

Table 2: Derived mass relation for $J^P = \frac{3}{2}^+$ particles. Here, m_i, m_j are the constituent quark masses of the respective quarks in the baryon, a_b, b are the parameters with the dimension of mass and m_0 is a scale factor equal to mass of the lightest quark, i.e $m_0 = m_u, m_d$.

The calculation of mass for proton is shown below:

$$\begin{aligned}
Mass_{(proton)} = & 0.43(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{2}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d} - \frac{3}{4m_u m_u}]) + \\
& 0.022(a_b + m_u + m_u + m_d + (-\frac{bm_0^2}{3}[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d}])) + \\
& 0.003(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{2}})[-\frac{3}{4m_u m_u} - \frac{1}{4m_u m_u} + \frac{1}{m_u m_d}]) + \\
& 0.142(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}})[-\frac{3}{4m_u m_u} + \frac{1}{4m_u m_u} - \frac{1}{m_u m_d}]) + \\
& 0.014(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{12}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d} + \frac{3}{4m_u m_u}] + \\
& (\frac{1}{\sqrt{12}} - \frac{1}{\sqrt{6}})[-\frac{3}{4m_u m_u} + \frac{1}{4m_u m_u} - \frac{1}{m_u m_d}]) + \\
& 0.0023(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d} + \frac{3}{4m_u m_u}]) + \\
& 0.0035(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{2} - \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{12}})[\frac{3}{4m_u m_u}]) + \\
& 0.0014(a_b + m_u + m_u + m_d + \frac{bm_0^2}{3}(\frac{1}{\sqrt{5}} - \sqrt{\frac{3}{20}} + \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{20}})[\frac{1}{4m_u m_u} - \frac{1}{m_u m_d}])
\end{aligned} \tag{4.7}$$

The set of various combinations of Fock states ($|gg\rangle, |u\bar{u}g\rangle, |d\bar{d}g\rangle$) sea is same for all baryon octet and decuplet members but there probability distribution is different due to mass inherited from flavor leading to different values of unknown parameters (a_0, a_8, a_{10}, b_1) etc.

5. Estimation of hadron masses

The baryon masses are calculated in literature using various models and taking the inputs of constituent quark masses. The constituent quark masses are model based parameters, so we allow suitable range (in MeV) to them and tries to fit these parameters to the available octet and decuplet masses, using *mathematica 7.0*. Here, the input in the mass formulae are the coefficients calculated statistically from the statistical model assuming sea quark-gluon Fock states to be in specific flavor, spin and color states. The model parameters for proton (in MeV) can be defined as:

$$m_u = m_0 = 290, m_d = 340, b = 600, a_b = 220$$

Now substituting these values of parameters in equation (5.1), we determine the mass of proton in D model with all sea contributions,

$$Mass_{(proton)} = 937.6 MeV$$

The mass of proton can be calculated with other modifications such as in C model or with individual sea contributions. Similarly, the model parameters for other baryons in octet and decuplet are calculated and is shown in the form of specific range below. For example, the model parameters (in MeV) for Σ^{*0} in decuplet can be shown as:

$$m_u = m_0 = 260, m_d = 310, m_s = 450, b = 600, a_b = 200$$

The calculation procedure described above leads to the results for masses of other octet and decuplet baryons as presented in table 3 and 4. The set of parameters (in MeV), for octet, in our model are: $b = 600$ to 630 , $m_u = 250$ to 300 , $m_d = 300$ to 340 , $m_s = 450$ to 550 and $a_b = 200$ to 230 . The set of parameters (in MeV), for decuplet, in our model are: $b = 600$ to 640 , $m_u = 200$ to 260 , $m_d = 250$ to 330 , $m_s = 400$ to 500 and $a_b = 200$ to 240 . The masses of spin 1/2 and 3/2 particles are computed in both C and D model with different sea contributions and are shown in the table 3 and 4, respectively. The masses mentioned in above set are the effective masses of quarks bound within hadrons i.e constituent quark masses.

Particle $J^P = \frac{1}{2}^+$	C Model (MeV)		D Model (MeV) [13]		Data (MeV) [13]
	With scalar sea	With (scalar+tensor) sea	With (scalar+tensor) sea	With (scalar+vector +tensor) sea	
p	1044.48	1053.17	857.29	937.6	938.27
n	1036.47	1045.7	938.8	939.85	939.56
Λ	1175.6	1187.53	1061.27	1113.5	1115.683
Σ^+	1249.53	1261.37	1141.98	1183.67	1189.37
Σ^0	1261.67	1239.81	1115.83	1189.05	1192.642
Σ^-	1226.15	1237.78	1165.86	1196.6	1197.449
Ξ^0	1469.13	1469.13	1315.99	1314.89	1314.86
Ξ^-	1390.72	1403.2	1267.03	1321.21	1321.71

Table 3: Masses of octet particles

Particle $J^P = \frac{3}{2}^+$	D Model (MeV)		C Model (MeV)		Data (MeV) [13]
	With scalar sea	With (scalar+tensor) sea	With (scalar+tensor) sea	With (scalar+vector+tensor) sea	
Δ^{++}	1234.17	1207.44	1231.57	1230.62	1232.0
Δ^+	1234.87	1210.5	1231.87	1230.7	1232.0
Δ^0	1229.44	1206.91	1226.58	1231.45	1232.0
Δ^-	1236.67	1204.38	1233.52	1232.53	1232.0
Σ^{*+}	1385.67	1364.76	1383.04	1382.17	1382.8
Σ^{*0}	1385.56	1367.32	1383.88	1383.22	1383.7
Σ^{*-}	1391.33	1369.76	1388.62	1387.72	1387.2
Ξ^{*0}	1621.9	1523.58	1531.46	1531.09	1531.80
Ξ^{*-}	1539.2	1516.11	1536.7	1535.8	1535.0
Ω^-	1670.0	1650.7	1668.37	1668.04	1672.45

Table 4: Masses of decuplet particles

To check the validity of our results, we have checked Gellmann-okubo mass formula for octet, equal spacing rule for decuplet and electromagnetic mass splittings, with the results from our model. The values obtained from our model are shown below every formula.

1. $\frac{N+\Xi}{2} = \frac{3\Lambda+\Sigma}{4}$
1129.40 1132.39
2. $\Delta - \Sigma^* = \Sigma^* - \Xi^* = \Xi^* - \Omega$
150.69 147.87 136.95
3. $\Delta^+ - \Delta^{++} = n - p - (\Sigma^+ + \Sigma^- - 2\Sigma^0)$
0.08 0.08
4. $\Delta^0 - \Delta^+ = \Sigma^{*0} - \Sigma^{*+} = n - p$
0.75 1.05 2.25
5. $\Delta^- - \Delta^0 = \Sigma^{*-} - \Sigma^{*0} = \Xi^{*-} - \Xi^{*0} = n - p + (\Sigma^+ + \Sigma^- - 2\Sigma^0)$
1.08 4.5 4.34 4.42

6. Discussion and Conclusion

We have calculated the masses of octet and decuplet particles using the mass formulae consisting of constituent quark masses and spin-spin interaction terms in statistical approach which assumes sea to be an admixture of gluons and quark-antiquark pairs in addition to valence quarks. The detailed analysis is based on calculation of masses within different approaches namely C and D model and further analyzing them by including the individual contributions from scalar, vector and tensor sea. Here the sea with spin 0, 1, 2 is called scalar, vector and tensor sea, respectively. Model D find the probabilities of Fock states by suppressing the contribution of states with higher multiplicities. Model D is assumed to be a special case of Model C. To appreciate the importance and validity of the sea with spin, various sea contributions are presented in table 3 and 4. This individual analysis of various sea and their possible contributions shows the importance of scalar, vector and tensor sea in finding the masses of spin $\frac{1}{2}$ and spin $\frac{3}{2}$ particles. Here, to check the contribution from the pure scalar sea, the following assumptions were made: $a_0 \neq 0$ and $b_1, b_8, d_1, d_8 = 0$, for the vector: $b_1, b_8 \neq 0$ and $a_0, d_1, d_8 = 0$ and similarly for the tensor $d_1, d_8 \neq 0$ and $a_0, b_1, b_8 = 0$. For the case of the scalar plus tensor sea: $a_0, d_1, d_8 \neq 0$ and $b_1, b_8 = 0$, for decuplet and similarly for octet.

Here, we can see from table 3 and 4 the masses comes out to be very close to the data from D model in case of octet whereas from C model in case of decuplet, respectively. Also specific dominance of scalar plus tensor sea contribution over vector is seen in both the cases. Here, the coefficients associated with the scalar, vector and tensor sea contributions are linearly related to the masses of octet and decuplet with an effect of the interaction terms coming from $(a_b, b_{...})$.

Here, the total multiplicities have been obtained using their respective spin, color and flavor space. In case of octet, since D model is giving better match with experimental data of masses and that too, major contribution comes from scalar plus tensor sea that dominates the contribution from vector sea by 66%. Similarly, for decuplet, C model gives better match with experimental data and hence scalar plus tensor sea contribution dominates the vector sea by 99.5%. Hence, the calculation of masses for both $J^P = \frac{1}{2}^+, \frac{3}{2}^+$, scalar plus tensor sea contribution dominance can be easily seen. In general, the sea is found to be dynamic for the scalar and tensor in both octet and decuplet. Here, spin-spin interaction term is dominating for vectorial sea and hence it suppresses the overall contribution to the masses from vector sea.

It can be very well seen from table 3, that the results from contribution of scalar sea in C model is showing a deviation of 5% to 11% while combination of scalar-tensor contribution shows a deviation of 3% to 12% when compared with experimental values available for $\frac{1}{2}^+$ particles. On the other hand results from D model are deviating 0.03% to 0.47%, when total sea is contributing, which shows that masses from total sea is providing good match with PDG data. Similarly, it can be seen from table 4, for decuplet, that contribution from total sea in case of C model is providing a good match with experimental values available as compared to individual contribution from sea.

It is also interesting to note that the set of constituent quark masses are different for every particle in octet and decuplet. If we insist to take single set of quark masses to hold for all particles and vary these masses, the overall best fit to the hadron data is deviating more from the experimental data. Moreover, the particles with two or three heavy (strange) quarks leads to less or negligible contribution of spin-spin interaction term to the overall mass of particles and hence will be less significant.

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